Decoding Literal Equations by Undressing the Process

Yoel Rodríguez
Department of Natural Sciences, Hostos Community College of CUNY

Allison Gorr
Department of Mathematics, Hostos Community College of CUNY

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Many students find calculus-based physics to be a very difficult subject. There is no question that it really is! However, if student preparation could be improved before they face the challenges that accompany an introductory calculus-based physics course, this perception of difficulty could be lessened. To this end, we would like to improve student understanding of literal equations by addressing this prerequisite topic during the first week of class and continuing to reinforce it throughout the remainder of the course. Understanding this concept is essential for success in any undergraduate physics course. [1] We will use the steps of the *Decoding the Disciplines* model to decode the process of solving literal equations (Pace and Middendorf, 2004).

Step 1. What is the Bottleneck to Learning in this Class?

An introductory calculus-based physics course requires students to be proficient with solving literal equations (symbolic equations) to better understand physical quantities and concepts. Students often struggle to manipulate literal equations to solve for specific variables (which represent particular physical quantities), and, more importantly, to translate the mathematics into and out of science. In general, most students can handle literal equations involving just one or two variables (along with numbers). When the literal equation includes several variables, however, students struggle to solve for and isolate the designated variable. Many problems presented during a physics course are stated just in terms of variables so students are required to provide their answers in terms of variables.

Since many students still struggle with literal equations at the beginning of their physics courses, this struggle becomes a bottleneck, jeopardizing the development of other problem-solving skills (such as abstract analysis and visualization), their understanding of important physics concepts (such as dimensional analysis), and their ability to successfully complete the course. Successful completion of a calculus-based physics course is required for many Science, Technology, Engineering and Mathematics (STEM) majors, so the stakes are high.

[1] We focus this discussion on Dr. Rodríguez's calculus-based course, General Physics I, but most of the ideas about teaching literal equations would apply to other mathematics and science courses as well.

Step 2. How Does an Expert Do These Things?

When confronting a literal equation, the expert begins by keeping the goal in mind, which is to isolate the designated variable, and understands that variables could be thought of as just placeholders for numbers. The expert then follows a proper sequence of steps while being careful to complete all mathematical operations correctly. This takes both patience and fortitude. If one idea does not work, he or she tries something else.

More specifically, the expert will often begin by underlining or circling the desired variable on the paper (or blackboard). He or she then relies on a multitude of mathematical competencies developed over years of experience. In **Table 1**, we list several of these mathematical competencies that the expert has already mastered. For example, the expert knows how to use parentheses correctly. In **Table 1**, we use specific examples to illustrate the level of student understanding (unacceptable versus developing), compared to what an expert would do with the particular example. Proficiency with these competencies and other algebraic skills is critical for success.

In addition to the competencies listed in **Table 1**, the expert also understands that he or she should proceed by "undoing" the various mathematical operations in an appropriate order until the desired variable has been successfully isolated. Experts will, for example, (a) undo addition with subtraction and vice versa, (b) undo multiplication with division and vice versa, (c) undo grouping symbols, and (d) undo exponents and radicals.

Overall, an expert is careful when writing equations and all intermediate steps. He or she knows that neatness and organization matter. Clarity in writing will help him or her to better visualize various types of problems.

Step 3. How Can These Tasks Be Explicitly Modeled?

We can model the "undo" concept by using an analogy. Consider what happens when a person dresses and undresses: to dress, one first puts on their t-shirt and then their shirt. To undress, one does the reverse (takes off their shirt and then their t-shirt). This is analogous to what happens when working with literal equations. A literal equation has already been "dressed" with variables, operation symbols, grouping symbols, and so on. To solve the equation for a specific variable, one needs to "undress" the equation step by step. This means to undo operations (*i.e.*, undo addition and subtraction, undo multiplication and division, undo grouping symbols, and undo exponents and radicals) until the desired variable has been isolated.

Once students understand this "undo" concept, the instructor can demonstrate problems on the whiteboard (or blackboard) to reinforce these ideas. The instructor should emphasize both proper sequence and proper use of algebraic operations (as described in **Table 1**). Students can then be given worksheets of examples so they can practice, with instructor guidance, beginning with simple literal equations and then moving on to increasingly complex problems. Physics students should be encouraged to express their final answers in terms of variables and then substitute appropriate numerical values into the equations. Practice with real-world examples, for which such solving and substitution would have intuitive meaning, will help students to better understand literal equations and the underlying physics concepts.

Table 1 Decoding Expert Thinking

Competencies	Unacceptable	Developing	Proficient
Using Parentheses Appropriately	The exponent is not applied to the entire term within the parentheses. $(2xy)^2 \neq 2xy^2$	Closer to correct. $(2xy)^2 \neq 2x^2y^2$	The exponent is applied correctly. $(2xy)^2 = 4x^2y^2$
Using the Distributive Property of Multiplication Correctly	The outside number is not distributed to every term within the parentheses. $2(2x^2 + 5x + 3) \neq 4x^2 + 5x + 3$	Closer to correct. $2(2x^2 + 5x + 3) \neq 4x^2 + 10x + 3$	The outside number is distributed correctly. $2(2x^2 + 5x + 3) = 4x^2 + 10x + 6$
Avoiding Exponent Issues	The exponent is simply distributed over the parentheses in which terms are added. $(x^2 + y^2)^2 \neq x^4 + y^4$	Closer to correct. $(x^2 + y^2)^2 \neq x^4 + x^2y^2 + y^4$	Exponent is distributed correctly. $(x^2 + y^2)^2 = x^4 + 2x^2y^2 + y^4$
Avoiding Simplification Issues	The denominator is not divided into every term of the numerator correctly. $\frac{2x^4 + 3x^2 - x}{x} \neq 2x^3 + 3x^2$	Closer to correct. $\frac{2x^4 + 3x^2 - x}{x} \neq 2x^3 + 3x$	The denominator is divided into the numerator correctly. $\frac{2x^4 + 3x^2 - x}{x} = 2x^3 + 3x - 1$
Writing Fractions Clearly	Fractions are not written clearly and consistently in all cases. $1/2x \ vs. \ 1/(2x)$ $a + b/c + d$ $vs. \ (a + b)/(c + d)$	Able to write simple cases correctly. Understand: $1/2x \ means \ \frac{1}{2}x$ $and \ not \ \frac{1}{2x}$	Able to write all cases correctly. Know: $a + b/c + d = a + \frac{b}{c} + d$ Know: $(a + b)/(c + d) = \frac{a + b}{c + d}$
Writing Variables Clearly	Subscript and/or superscript notation is used inappropriately. Write $x5$ instead of x_5 Write $x2$ instead of x^2	Often able to use subscript and superscript notation correctly.	Always able to use subscript and superscript notation correctly. Use x_5 when want to label a variable. Use x^2 when want to square a variable.

Table 2 includes some examples of literal equations relevant to specific physics problems. Each literal equation could be solved for any of the variables included in the equation (see below). As these physics topics come up in the curriculum, the instructor can model how to solve the equations for other variables.

This type of explicit modeling can be implemented throughout the course with every topic that involves literal equations. This will help students better understand the physics behind the phenomena contained within a particular problem.

Table 2 Examples of Literal Equations Used in Physics Problems

Difficulty	Phenomenon | Topic	Literal Equation	Where
Easy	Motion of a particle in one dimension. Used to determine the *distance* traveled by the particle between initial and final positions.	$s = vt$	v, s, and t represent speed, distance, and time, respectively.
Medium	Motion of a particle on an inclined plane with kinetic friction. Used to determine the *normal force* and the *acceleration* of the particle sliding down the inclined plane.	$n = mg\cos\alpha$ $a_x = g(\sin\alpha - \mu_k\cos\alpha)$	n, m, g, and α represent the normal force, the mass of the particle, the acceleration due to gravity and the angle of the inclined plane, respectively. a_x, μ_k, g, and α represent the acceleration of the particle, the coefficient of kinetic friction between the particle and the inclined plane surface, the acceleration due to gravity, and the angle of the inclined plane, respectively.
Challenge	Motion of a crate by pulling it upward with a rope at an angle α with respect to the horizontal. Used to determine the *tension in the rope* so the crate moves with *constant velocity*.	$T = \frac{\mu_k mg}{(\cos\alpha + \mu_k\sin\alpha)}$	T, μ_k, m, g, and α represent the tension force on the rope, the coefficient of kinetic friction between the crate and the surface, the mass of the crate, the acceleration due to gravity, and the angle of the inclined plane, respectively.

Step 4. How Will Students Practice These Skills and Get Feedback?

Students will practice solving literal equations through instructor-led demonstrations, class discussion, group worksheets, and individual assignments (see **Appendix I**). All problems will be connected to the appropriate physics concepts and gradually increase in difficulty. Feedback will come in written form through the grading of assignments and in verbal form through class discussion.

Professor Rodríguez provides a detailed example below of how the study of projectile motion can be an excellent way for students to practice solving literal equations in a physics context. The exercise incorporates all six levels of Bloom's taxonomy, an important framework for focusing on higher-order thinking skills. He plans to include this projectile motion exercise in his calculus-based physics course next semester (see **Table 3** for a visual aid and **Appendix II** for details about the physics equations actually used in the exercise).

The main steps involved in the exercise are listed below:

1. How could you describe physically and mathematically the motion of a ball? What do you think affects the motion of a ball? (Bloom's: Describing, Identifying, Reproducing)
2. Provide equation(s) to describe the ball's motion. (Bloom's: Identifying, Describing, Applying)
3. Relate variables to the phenomena. (Bloom's: Translating, Identifying, Analyzing)
4. Practice by generating different situations. Solve literal equations for different physical quantities. How do variables interact? Order of Operations: 'undo' operations to solve for different physical quantities. (Bloom's: Evaluating)
5. Side-visit with solving quadratic equations. (Bloom's: Evaluating)
6. Graph your results. (Bloom's: Creating, Evaluating)
7. Test extremes. Exploring equation effects. (Bloom's: Applying, Evaluating)
8. Interpret your results. (Bloom's: Translating, Evaluating, Collecting, Explaining)

We want students to see that successfully solving literal equations is an integral part of understanding important concepts. The skill does not just exist in isolation, separate from their other mathematical and scientific pursuits.

Table 3 Using Free Falling and Projectile Motions to Understand Literal Equations From One Dimension to Two Dimensions

Teacher: __Prof. Rodríguez__ Course: __PHY 210__ Grade: __________ Unit: ___Chapters 2 and 3___ Timeframe: ___3 hours___

1. How could you describe physically and mathematically the motion of a ball? How do different variables affect the motion of a ball?

Bloom's: Describing, Identifying, Reproducing

Feedback Criteria: see **Step 5**

2. Provide equation(s) to describe the ball's motion.

Bloom's: Identifying, Describing, Applying

Feedback Criteria: see **Step 5**

3. Relate variables to the phenomena.

Bloom's: Translating, Identifying, Analyzing

Feedback Criteria: see **Step 5**

4. Practice by generating different situations. Solve literal equations for different physical quantities. How do variables interact? Order of Operations: 'undo' operations to solve for different physical quantities.

Bloom's: Evaluating

Feedback Criteria: see **Step 5**

Bottleneck

Students struggle to solve literal equations.

Decode Expert Thinking

An expert can use physics phenomenon to better understand how to solve and interpret literal equations.

Modeling Metaphor

Solving literal equations is like when one dresses and undresses. To dress, one first puts on their t-shirt and then their shirt. To undress one does the reverse: one takes off their shirt and then their t-shirt (see **Step 3**).

Bloom's Used:
Describing, Identifying, Reproducing, Translating, Analyzing Evaluating, Applying, Collecting, Explaining, Creating

5. Side-visit with solving quadrating.

Bloom's: Applying, Evaluating

Feedback Criteria: see **Step 5**

6. Graph your results

Bloom's: Creating, Evaluating

Feedback Criteria: see **Step 5**

7. Test extremes. Exploring equation effects.

Bloom's: Applying, Evaluating

Feedback Criteria: see **Step 5**

8. Interpret your results.

Bloom's: Translating, Evaluating, Collecting, Explaining

Feedback Criteria: see **Step 5**

Step 5. What Will Motivate the Students?

Students in a calculus-based physics course are usually STEM majors, who can already appreciate the need to further develop their mathematical skills. We can add to this foundation through class discussion of additional applications of literal equations in the careers they plan to pursue. We can also emphasize the need for patience and persistence. Another possibility is to share the interesting videos that were part of the Collaborative Curriculum Revision Project (CCRP) program. We refer specifically to Austin's Butterfly video and the two growth-versus-fixed mindset videos.[2,3]

To prepare for these ongoing motivational discussions, it is useful to know the skill level and the educational mindset each student already has. To assess these items, we plan to give a three-part assessment on the first day of class. The assessment will include a series of questions about students' class expectations (and emotional bottlenecks), their current skill-levels pertaining to literal equations, and their learning orientations (*i.e.*, growth or fixed mindset, identified through a mindset quiz) (see **Tables AIII.1 and AIII.2 in Appendix III**).

When students feel frustrated, we can offer encouragement and useful feedback to motivate them to do their best. All praise should encourage a growth mindset, rather than a fixed mindset. We want students to see that their abilities and intelligence can be developed over time with effort and persistence. Some useful statements are listed below:

- You're on the right track - you just need to be more careful with [insert specific error].
- It's easier to remember the rules and steps if your writing is more neat and organized.
- Let's try some more medium level problems, before tackling the harder ones.
- You're doing much better! I'm confident you'll get the next one right.
- Great job! You must have worked really hard. Let's try a more challenging problem.
- I like how you tried all kinds of strategies until you got it right.
- It seems like you are not ready for this topic YET, but you WILL be able to do it.
- Keep working hard, but more efficiently.
- Keep in mind these three important words in learning: Understanding, Visualizing and Practicing.

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[2] Austin's Butterfly video: https://www.youtube.com/watch?v=dOSiU42P8Gc, April 2019
Growth mindset video 1: https://www.youtube.com/watch?v=KUWn_TJTrnU, April 2019
Growth mindset video 2: https://www.youtube.com/watch?v=J-swZaKN2Ic, April 2019

[3] In her book and videos, researcher Carol Dweck defines both the growth mindset and the fixed mindset. In a fixed mindset, people believe their intelligence and talents are fixed traits that cannot change. In a growth mindset, people believe their intelligence and talents can grow with effort and persistence. See the growth mindset videos identified in the previous footnote (2).

Step 6. How Well are the Students Mastering These Learning Tasks?

We will begin applying our strategy to decode literal equations during the next academic year. We expect this strategy to help students better understand and solve literal equations. In addition to class discussion, we intend to use the following types of assessments to evaluate the strengths and weaknesses of the strategy. We are prepared to modify and adapt our strategy as outcomes suggest.

1. **Formative assessments**
   We will give these assessments throughout the course to provide quick feedback to both the instructor and the students. These assessments will show the instructor which ideas may need to be reviewed before moving on to new topics. For example, we can use "pop quizzes" (*i.e.*, short tests given without prior warning during the first 5 minutes of class) to check if students understand the previous lesson. We can then utilize *Peer Instruction* and *Just-in-Time Teaching* approaches to correct deficiencies and aid student learning during the rest of the class (Lasry, Mazur and Watkins, 2008, and Formica, Easley, and Spraker, 2010).

2. **Summative assessments**
   We will administer these assessments in the middle and at the end of the semester to measure student knowledge of course content and to highlight cases of significant improvement. We plan for these assessments to be the midterm exam and the final exam.

The implementation and assessment of this decoding strategy will provide useful feedback for everyone involved. We hope to help students become better critical thinkers and more emotionally prepared to face the challenges of their disciplines as they move forward in their academic and personal lives.

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Appendix I | Literal Equations Worksheet

Professor Rodríguez intends to use the following worksheet in his physics course. Based on years of experience, he knows that most incoming students have trouble with literal equations, which is a significant impediment to success in his course. The purpose of the worksheet is to lead students, step by step, through the process of solving specific examples. Each step will be thoroughly discussed and demonstrated in class. Other similar worksheets will continue to be used throughout the course. Students will have opportunities to work together and individually.

Solving Literal Equations, Step by Step

Example 1

Solve $℃ = \frac{5}{9}(℉ - 32)$ for ℉ (**Fahrenheit and Celsius Equation**), where ℉ and ℃ are the Fahrenheit and Celsius Temperatures, respectively.

Multiply both sides of the equation by 9 to clear the fraction

$$(9)℃ = \frac{5}{9}(℉ - 32)\ (9)$$

Simplify

$$(9)℃ = \frac{5}{\cancel{9}}(℉ - 32)\ \cancel{(9)}$$

$$(9)℃ = 5(℉ - 32)$$

Divide both sides of the equation by 5 to isolate ℉

$$\frac{9}{5}℃ = \cancel{5}(℉ - 32)\frac{1}{\cancel{5}}$$

Simplify

$$\frac{9}{5}℃ = (℉ - 32)$$

Add 32 to both sides of the equation to get ℉ **isolated**

$$\frac{9}{5}℃ + 32 = (℉ - \cancel{32} + \cancel{32})$$

So ℉ **is isolated and the solution is**

$$℉ = \frac{9}{5}℃ + 32$$

Example 2

Solve $v_x = v_{0x} + a_x t$ for t (**Constant Acceleration Motion in One Direction**), where v_x is the velocity of the particle at any time, v_{0x} is the initial velocity of the particle, a_x is the acceleration of the particle and t represents time.

Subtract v_{ox} from both sides to get $a_x t$ alone

$$v_x - v_{0x} = v_{0x} + a_x t - v_{0x}$$

Simplify

$$v_x - v_{0x} = \cancel{v_{0x}} + a_x t - \cancel{v_{0x}}$$

$$v_x - v_{0x} = a_x t$$

Divide by a_x both sides of the equations to isolate t

$$\frac{v_x - v_{0x}}{a_x} = \frac{a_x t}{a_x}$$

Simplify

$$\frac{v_x - v_{0x}}{a_x} = \frac{\cancel{a_x} t}{\cancel{a_x}}$$

So t is isolated and the solution is

$$t = \frac{v_x - v_{0x}}{a_x}$$

Example 3

Solve $W = \frac{1}{2}\, m v_2^2 - \frac{1}{2}\, m v_1^2$ for v_1 (**Work-Energy Theorem**), where W stands for total work, m is the mass of the particle, and v_1 and v_2, represent the initial and final speeds of the particle, respectively.

Multiply both sides of the equation by 2 to clear the fraction

$$(2)W = \left(\frac{1}{2}\, m v_2^2 - \frac{1}{\cancel{2}}\, m v_1^2\right)(2)$$

Simplify

$$(2)W = \left(\frac{1}{\cancel{2}}\, m v_2^2 - \frac{1}{\cancel{2}}\, m v_1^2\right)(\cancel{2})$$

$$(2)W = m v_2^2 - m v_1^2$$

Subtract $m v_2^2$ from both sides to get $m v_1^2$ alone

$$(2)W - mv_2^2 = mv_2^2 - mv_1^2 - mv_2^2$$

Simplify

$$(2)W - mv_2^2 = \cancel{mv_2^2} - mv_1^2 - \cancel{mv_2^2}$$

$$(2)W - mv_2^2 = -mv_1^2$$

Multiply by -1 both sides of the equation to make positive the term mv_1^2

$$[(2)W - mv_2^2](-1) = (-mv_1^2)(-1)$$

$$-(2)W + mv_2^2 = mv_1^2$$

$$mv_2^2 - 2W = mv_1^2$$

Divide by m both sides of the equations to isolate v_1^2

$$\left(\frac{mv_2^2 - 2W}{m}\right) = \frac{mv_1^2}{m}$$

Simplify

$$\left(\frac{mv_2^2 - 2W}{m}\right) = \frac{\cancel{m}v_1^2}{\cancel{m}}$$

$$\left(\frac{mv_2^2 - 2W}{m}\right) = v_1^2$$

To isolate, v_1, take the square root of both sides of the equation

$$\sqrt{\left(\frac{mv_2^2 - 2W}{m}\right)} = \sqrt{v_1^2}$$

So v_1 is isolated and the solution is

$$v_1 = \pm\sqrt{\left(\frac{mv_2^2 - 2W}{m}\right)}$$

Appendix II | Free Fall and Projectile Motions (used with Table 3)

Motion in One Dimension | Free Fall Motion (Air Resistance is Neglected)

The position of a particle as a function of time in free fall motion is given by

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad (1)$$

where y, y_0, v_{0y}, t, and a_y represent position of the particle at any time, the initial position, the initial velocity, time, and the acceleration due to gravitational force ($g = 9.80\ {}^{m}/_{s^2}$), respectively. The y-component of the acceleration is $a_y = -g$ if the coordinate system is chosen positive upward.

Motion in Two Dimensions | Projectile Motion (Air Resistance is Neglected)

In the x-direction, the motion occurs at constant velocity and in the y-direction, the motion occurs with constant acceleration. The motion in the x- and the y- directions occur independently. The position of a particle in the x-direction as a function of time in projectile motion is given by

$$x = x_0 + v_{0x}t = x_0 + v_0\, cos\, \alpha_0\, t \quad (2)$$

where x, x_0, v_{0x}, t, α_0 represent position at any time, initial position, the x-component of the initial velocity, time, and the angle of the initial velocity with respect to the horizontal, respectively. The x-component of the initial velocity is given by $v_{0x} = v_0 \cos \alpha_0$.

The y-position of a particle as a function of time in projectile motion is given by

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad (3)$$

where y, y_0, v_{0y}, t, and a_y represent position at any time, initial position, the y-component of the initial velocity, time, and acceleration due to gravitational force ($g = 9.80\ {}^{m}/_{s^2}$), respectively. If the coordinate system is chosen positive upward, $a_y = -g$. The y-component of the initial velocity is given by $v_{0y} = v_0 \sin \alpha_0$.

The trajectory's shape of the motion of the particle as a function of x and y can be obtained by eliminating t from Eqs. 2 and 3.

$$y = (tan\, \alpha_0)x - \frac{g}{2v_0^2(cos\, \alpha_0)^2}x^2 \quad (4)$$

Equation 4 can be written as

$$y = bx - cx^2 \quad (5)$$

where $b = \tan \alpha_0$ and $c = \frac{g}{2v_0^2(\cos \alpha_0)^2}$ are constants. Equation 5 represents a parabola.

For further details about free fall and projectile motions, we recommend reviewing a textbook such as University Physics by Young and Freedman, 2016.

Appendix III | First-Day-of-Class Assessment

This three-part assessment will help identify students' class expectations (and emotional bottlenecks), their current skill-levels pertaining to literal equations, and their learning orientations (*i.e.*, growth or fixed mindset). This assessment will be given on the first day of class and will be used to address specific student needs.

Part I: Class Expectations (Adapted from Mazur, 1994)

To assess class expectations, we will ask the following questions:

1. In addition to hard work, what do you think it takes to do well in this course?
2. What do you expect to learn from this course?
3. What do you expect to do with this new knowledge?
4. What do you expect the lectures to do for you?
5. What do you expect the book to do for you?
6. Do you review the topics of the previous lecture before coming to class?
   Always Frequently Sometimes Never
7. About how many hours per week do you think it will take to learn all you need to know in this course? Include everything: lecture, recitation, lab, homework, quizzes, exams, etc.
   ______ hours/week

Part II: Skill-Level

Students' ability to do these problems, including the work they show along the way, will illustrate how well they understand the concept of solving literal equations.

1. Solve the following equation for *v*

$$\mu_k m_A g d = -m_B g d + \frac{1}{2}\left(m_A + m_B + \frac{I}{R^2}\right)v^2$$

2. Newton's law of universal gravitation is represented by $F = G\frac{Mm}{r^2}$

   Here *F* is the magnitude of the gravitational force exerted by one small object on another, *M* and *m* are the masses of the objects, and *r* is the distance between them. Force has the units $kg \cdot m/s^2$. What are the *SI units* of the proportionality constant *G*? SI means International System (abbreviated from the French Système International).

Part III: Mindset Quiz (Adapted from Shipley website and Diehl website; see **References**)

We will ask students to complete the following mindset quiz by checking the appropriate boxes. See **Table AIII.1** for the quiz and **Table AIII.2** for the scoring rubric.

Table AIII.1 Mindset Quiz

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. Your intelligence is something very basic about you that you cannot change very much.				
2. No matter how much intelligence you have, you can always change it quite a bit.				
3. You can always substantially change how intelligent you are.				
4. You are a certain kind of person, and there is not much that can be done to really change that.				
5. You can always change basic things about the kind of person you are.				
6. Music talent can be learned by anyone.				
7. Only a few people will be truly good at sports – you have to be "born with it."				
8. Math is much easier to learn if you are male or maybe come from a culture who values math.				
9. The harder you work at something, the better you will be at it.				
10. No matter what kind of person you are, you can always change substantially.				
11. Trying new things is stressful for me and I avoid it.				
12. Some people are good and kind, and some are not – it is not often that people change.				
13. I appreciate when parents, coaches, teachers give me feedback about my performance.				
14. I often get angry when I get feedback about my performance.				
15. All human beings without a brain injury or birth defect are capable of the same amount of learning.				
16. You can learn new things, but you cannot really change how intelligent you are.				
17. You can do things differently, but the important parts of who you are cannot really be changed.				
18. Human beings are basically good, but sometimes make terrible decisions.				
19. An important reason why I do my school work is that I like to learn new things.				
20. Truly smart people do not need to try hard.				

Table AIII.2 Mindset Quiz Rubric

To score the quiz, we circle the number in the box that matches each answer and then total the scores.

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. Ability mindset – fixed	0	1	2	3
2. Ability mindset – growth	3	2	1	0
3. Ability mindset – growth	3	2	1	0
4. Personality/character mindset – fixed	0	1	2	3
5. Personality/character mindset – growth	3	2	1	0
6. Ability mindset – growth	3	2	1	0
7. Ability mindset – fixed	0	1	2	3
8. Ability mindset – fixed	0	1	2	3
9. Ability mindset – growth	3	2	1	0
10. Personality/character mindset - growth	3	2	1	0
11. Ability mindset – fixed	0	1	2	3
12. Personality/character mindset – fixed	0	1	2	3
13. Ability mindset –growth	3	2	1	0
14. Ability mindset – fixed	0	1	2	3
15. Ability mindset – growth	3	2	1	0
16. Ability mindset – fixed	0	1	2	3
17. Personality/character mindset – fixed	0	1	2	3
18. Personality/character mindset –growth	3	2	1	0
19. Ability mindset – growth	3	2	1	0
20. Ability mindset – fixed	0	1	2	3
Total				
Grand Total				

Results Legend:

Strong Growth Mindset = 45 – 60 points

Growth Mindset with some Fixed ideas = 34 – 44 points

Fixed Mindset with some Growth ideas = 21 – 33 points

Strong Fixed Mindset = 0 – 20 points